Exercise 16.1

**Question 1:**
Find the values of the letters in the following and give reasons for the steps involved.

\[
\begin{array}{c}
3 \quad A \\
+ \quad 2 \quad 5 \\
\_ \quad B \quad 2
\end{array}
\]

**Answer:**
The addition of \( A \) and 5 is giving 2 i.e., a number whose ones digit is 2. This is possible only when digit \( A \) is 7. In that case, the addition of \( A \) (7) and 5 will give 12 and thus, 1 will be the carry for the next step. In the next step, \( 1 + 3 + 2 = 6 \)
Therefore, the addition is as follows.

\[
\begin{array}{c}
3 \quad 7 \\
+ \quad 2 \quad 5 \\
\_ \quad 6 \quad 2
\end{array}
\]

Clearly, \( B \) is 6.
Hence, \( A \) and \( B \) are 7 and 6 respectively.

**Question 2:**
Find the values of the letters in the following and give reasons for the steps involved.

\[
\begin{array}{c}
4 \quad A \\
+ \quad 9 \quad 8 \\
\_ \quad C \quad B \quad 3
\end{array}
\]

**Answer:**
The addition of \( A \) and 8 is giving 3 i.e., a number whose ones digit is 3. This is possible only when digit \( A \) is 5. In that case, the addition of \( A \) and 8 will give 13 and thus, 1 will be the carry for the next step. In the next step, \( 1 + 4 + 9 = 14 \)
Therefore, the addition is as follows.
4 5  
+ 9 8  
---  
14 3  

Clearly, B and C are 4 and 1 respectively.  
Hence, A, B, and C are 5, 4, and 1 respectively.  

**Question 4:**  
Find the values of the letters in the following and give reasons for the steps involved.  

```
A  B  
+ 3  7  
---  
6  A  
```

**Answer:**  
The addition of A and 3 is giving 6. There can be two cases.  
(1) First step is not producing a carry  
In that case, A comes to be 3 as 3 + 3 = 6. Considering the first step in which the addition of B and 7 is giving A (i.e., 3), B should be a number such that the units digit of this addition comes to be 3. It is possible only when B = 6. In this case, A = 6 + 7 = 13. However, A is a single digit number. Hence, it is not possible.  
(2) First step is producing a carry  
In that case, A comes to be 2 as 1 + 2 + 3 = 6. Considering the first step in which the addition of B and 7 is giving A (i.e., 2), B should be a number such that the units digit of this addition comes to be 2. It is possible only when B = 5 and 5 + 7 = 12.  

```
2  5  
+ 3  7  
---  
6  2  
```

Hence, the values of A and B are 2 and 5 respectively.
**Question 5:**
Find the values of the letters in the following and give reasons for the steps involved.

\[
\begin{array}{c}
A \\ B \\
\times 3 \\
\hline
C \\ A \\ B \\
\end{array}
\]

**Answer:**
The multiplication of 3 and B gives a number whose ones digit is B again.
Hence, B must be 0 or 5.
Let B be 5.
Multiplication of first step = 3 \times 5 = 15
1 will be a carry for the next step.
We have, 3 \times A + 1 = CA
This is not possible for any value of A.
Hence, B must be 0 only. If B = 0, then there will be no carry for the next step.
We should obtain, 3 \times A = CA
That is, the one's digit of 3 \times A should be A. This is possible when A = 5 or 0.
However, A cannot be 0 as AB is a two-digit number.
Therefore, A must be 5 only. The multiplication is as follows.

\[
\begin{array}{c}
50 \\
\times 3 \\
\hline
150 \\
\end{array}
\]

Hence, the values of A, B, and C are 5, 0, and 1 respectively.

**Question 6:**
Find the values of the letters in the following and give reasons for the steps involved.

\[
\begin{array}{c}
A \\ B \\
\times 5 \\
\hline
C \\ A \\ B \\
\end{array}
\]

**Answer:**
The multiplication of $B$ and 5 is giving a number whose ones digit is $B$ again. This is possible when $B = 5$ or $B = 0$ only.

In case of $B = 5$, the product, $B \times 5 = 5 \times 5 = 25$

2 will be a carry for the next step.

We have, $5 \times A + 2 = CA$, which is possible for $A = 2$ or 7

The multiplication is as follows.

\[
\begin{array}{c c c c}
25 & & 75 \\
\times 5 & & \times 5 \\
\hline
125 & & 375 \\
\end{array}
\]

If $B = 0$,

$B \times 5 = B \Rightarrow 0 \times 5 = 0$

There will not be any carry in this step.

In the next step, $5 \times A = CA$

It can happen only when $A = 5$ or $A = 0$

However, $A$ cannot be 0 as $AB$ is a two-digit number.

Hence, $A$ can be 5 only. The multiplication is as follows.

\[
\begin{array}{c c c c}
50 & & \\
\times 5 & & \\
\hline
250 & & \\
\end{array}
\]

Hence, there are 3 possible values of $A$, $B$, and $C$.

(i) 5, 0, and 2 respectively

(ii) 2, 5, and 1 respectively

(iii) 7, 5, and 3 respectively

**Question 7:**

Find the values of the letters in the following and give reasons for the steps involved.

\[
\begin{array}{c c c c}
A & B & \times 6 \\
\times 6 & & \\
B & B & B \\
\end{array}
\]
Answer:
The multiplication of 6 and B gives a number whose one’s digit is B again.
It is possible only when B = 0, 2, 4, 6, or 8
If B = 0, then the product will be 0. Therefore, this value of B is not possible.
If B = 2, then B \times 6 = 12 and 1 will be a carry for the next step.
6A + 1 = BB = 22 \Rightarrow 6A = 21 and hence, any integer value of A is not possible.
If B = 6, then B \times 6 = 36 and 3 will be a carry for the next step.
6A + 3 = BB = 66 \Rightarrow 6A = 63 and hence, any integer value of A is not possible.
If B = 8, then B \times 6 = 48 and 4 will be a carry for the next step.
6A + 4 = BB = 88 \Rightarrow 6A = 84 and hence, A = 14. However, A is a single digit number. Therefore, this value of A is not possible.
If B = 4, then B \times 6 = 24 and 2 will be a carry for the next step.
6A + 2 = BB = 44 \Rightarrow 6A = 42 and hence, A = 7
The multiplication is as follows.

\[
\begin{array}{c}
  \text{7} \quad \text{4} \\
  \times \quad \text{6} \\
  \hline
  \text{4} \quad \text{4} \quad \text{4}
\end{array}
\]

Hence, the values of A and B are 7 and 4 respectively.

Question 8:
Find the values of the letters in the following and give reasons for the steps involved.

\[
\begin{array}{c}
  \text{A} \quad \text{1} \\
  \text{+ 1 B} \\
  \hline
  \text{B} \quad \text{0}
\end{array}
\]

Answer:
The addition of 1 and B is giving 0 i.e., a number whose ones digits is 0. This is possible only when digit B is 9. In that case, the addition of 1 and B will give 10 and thus, 1 will be the carry for the next step. In the next step, 1+A+1=B
Clearly, A is 7 as \(1 + 7 + 1 = 9 = B\)

Therefore, the addition is as follows.

\[
\begin{array}{c}
7 \\
+ 1 \\
\hline
9 \\
\end{array}
\]

Hence, the values of A and B are 7 and 9 respectively.

**Question 9:**

Find the values of the letters in the following and give reasons for the steps involved.

\[
\begin{array}{c}
2 \ A \ B \\
+ A \ B \ 1 \\
\hline
B \ 1 \ 8 \\
\end{array}
\]

**Answer:**

The addition of B and 1 is giving 8 i.e., a number whose ones digits is 8. This is possible only when digit B is 7. In that case, the addition of B and 1 will give 8. In the next step,

\(A + B = 1\)

Clearly, A is 4.

\(4 + 7 = 11\) and 1 will be a carry for the next step. In the next step,

\(1 + 2 + A = B\)

\(1 + 2 + 4 = 7\)

Therefore, the addition is as follows.

\[
\begin{array}{c}
2 \ 4 \ 7 \\
+ 4 \ 7 \ 1 \\
\hline
7 \ 1 \ 8 \\
\end{array}
\]

Hence, the values of A and B are 4 and 7 respectively.
**Question 10:**

Find the values of the letters in the following and give reasons for the steps involved.

\[
\begin{array}{c}
1 \quad 2 \quad A \\
+ \quad 6 \quad A \quad B \\
\hdashline
\quad A \quad 0 \quad 9 \\
\end{array}
\]

**Answer:**

The addition of A and B is giving 9 i.e., a number whose ones digit is 9. The sum can be 9 only as the sum of two single digit numbers cannot be 19. Therefore, there will not be any carry in this step.

In the next step, \(2 + A = 0\)

It is possible only when \(A = 8\)

\(2 + 8 = 10\) and 1 will be the carry for the next step.

\[1+1+6=A\]

Clearly, A is 8. We know that the addition of A and B is giving 9. As A is 8, therefore, B is 1.

Therefore, the addition is as follows.

\[
\begin{array}{c}
128 \\
+ \quad 681 \\
\hdashline
\quad 809 \\
\end{array}
\]

Hence, the values of A and B are 8 and 1 respectively.
Question 1:
If 21y5 is a multiple of 9, where y is a digit, what is the value of y?

Answer:
If a number is a multiple of 9, then the sum of its digits will be divisible by 9.
Sum of digits of 21y5 = 2 + 1 + y + 5 = 8 + y
Hence, 8 + y should be a multiple of 9.
This is possible when 8 + y is any one of these numbers 0, 9, 18, 27, and so on ...
However, since y is a single digit number, this sum can be 9 only. Therefore, y should be 1 only.

Question 2:
If 31z5 is a multiple of 9, where z is a digit, what is the value of z?
You will find that there are two Answers for the last problem. Why is this so?

Answer:
If a number is a multiple of 9, then the sum of its digits will be divisible by 9.
Sum of digits of 31z5 = 3 + 1 + z + 5 = 9 + z
Hence, 9 + z should be a multiple of 9.
This is possible when 9 + z is any one of these numbers 0, 9, 18, 27, and so on ...
However, since z is a single digit number, this sum can be either 9 or 18. Therefore, z should be either 0 or 9.

Question 3:
If 24x is a multiple of 3, where x is a digit, what is the value of x?
(Since 24x is a multiple of 3, its sum of digits 6 + x is a multiple of 3; so 6 + x is one of these numbers: 0, 3, 6, 9, 12, 15, 18.... But since x is a digit, it can only be that 6 + x = 6 or 9 or 12 or 15. Therefore, x = 0 or 3 or 6 or 9. Thus, x can have any of four different values)

Answer:
Since 24x is a multiple of 3, the sum of its digits is a multiple of 3.
Sum of digits of 24x = 2 + 4 + x = 6 + x
Hence, \(6 + x\) is a multiple of 3.
This is possible when \(6 + x\) is any one of these numbers 0, 3, 6, 9, and so on ...
Since \(x\) is a single digit number, the sum of the digits can be 6 or 9 or 12 or 15 and thus, the value of \(x\) comes to 0 or 3 or 6 or 9 respectively.
Thus, \(x\) can have its value as any of the four different values 0, 3, 6, or 9.

**Question 4:**
If \(31z5\) is a multiple of 3, where \(z\) is a digit, what might be the values of \(z\)?

**Answer:**
Since \(31z5\) is a multiple of 3, the sum of its digits will be a multiple of 3.
That is, \(3 + 1 + z + 5 = 9 + z\) is a multiple of 3.
This is possible when \(9 + z\) is any one of 0, 3, 6, 9, 12, 15, 18, and so on ...
Since \(z\) is a single digit number, the value of \(9 + z\) can only be 9 or 12 or 15 or 18 and thus, the value of \(x\) comes to 0 or 3 or 6 or 9 respectively.
Thus, \(z\) can have its value as any one of the four different values 0, 3, 6, or 9.